

Chapter 9 - Day 1

Summation Rules

$$\textcircled{1} \sum_{k=1}^n c = nc$$

$$\textcircled{2} \sum_{k=1}^n (c a_k) = c \sum_{k=1}^n a_k$$

$$\textcircled{3} \sum_{k=1}^n (a_k \pm b_k) = \sum_{k=1}^n a_k \pm \sum_{k=1}^n b_k$$

Summation Formulas

$$\textcircled{1} \sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\textcircled{2} \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

Ex: Evaluate $\sum_{k=1}^7 (3k+8)$

$$\begin{aligned}\sum_{k=1}^7 (3k+8) &= \sum_{k=1}^7 3k + \sum_{k=1}^7 8 \\ &= 3 \sum_{k=1}^7 k + \sum_{k=1}^7 8 \\ &= 3 \left(\frac{7(8)}{2} \right) + 7(8) \\ &= 3(28) + 56 \\ &= \boxed{140}\end{aligned}$$

Ex: Evaluate $\sum_{k=1}^9 (5k^2 + 7k + 2)$

$$\sum_{k=1}^9 (5k^2 + 7k + 2) = \sum_{k=1}^9 5k^2 + \sum_{k=1}^9 7k + \sum_{k=1}^9 2$$

$$= 5 \sum_{k=1}^9 k^2 + 7 \sum_{k=1}^9 k + \sum_{k=1}^9 2$$

$$= 5 \left(\frac{9(10)(19)}{6} \right) + 7 \left(\frac{9(10)}{2} \right) + 9(2)$$

$$= 5(285) + 7(45) + 18$$

$$= \boxed{1758}$$

Ex: Evaluate $\sum_{k=5}^{10} (k+7)$

We only know formulas for $k=1 \dots n$

We only want the 5th - 10th terms.

$$\sum_{k=5}^{10} (k+7) = \sum_{k=1}^{10} (k+7) - \sum_{k=1}^4 (k+7)$$

$$= \sum_{k=1}^{10} k + \sum_{k=1}^{10} 7 - \left[\sum_{k=1}^4 k + \sum_{k=1}^4 7 \right]$$

$$= \frac{10(11)}{2} + 10(7) - \left[\frac{4(5)}{2} + 4(7) \right]$$

$$= 55 + 70 - 10 - 28$$

$$= \boxed{87}$$

Ex: Evaluate $\sum_{k=3}^{100} (1+7k)$

$$\begin{aligned}\sum_{k=3}^{100} (1+7k) &= \sum_{k=1}^{100} (1+7k) - \sum_{k=1}^2 (1+7k) \\ &= \sum_{k=1}^{100} 1 + 7 \sum_{k=1}^{100} k - \left[\sum_{k=1}^2 1 + 7 \sum_{k=1}^2 k \right] \\ &= 100(1) + 7 \left(\frac{100(101)}{2} \right) - \left[2(1) + 7 \left(\frac{2(3)}{2} \right) \right] \\ &= 100 + 7(5050) - 2 - 7(3) \\ &= \boxed{35427}\end{aligned}$$

Ex: Evaluate $1 + 5 + 10 + 15 + 20 + \dots + 245$

$5k$

$$= 1 + \sum_{k=1}^{49} 5k$$

$$= 1 + 5 \sum_{k=1}^{49} k$$

$$= 1 + 5 \left(\frac{49(50)}{2} \right)$$

$$= 1 + 5(1225)$$

$$= \boxed{6126}$$

Ex: Evaluate the sum

$$16 + 20 + 24 + \dots + 104$$

$4k$

$$= \sum_{k=4}^{26} 4k$$

$$= 4 \sum_{k=4}^{26} k$$

$$= 4 \left(\sum_{k=1}^{26} k - \sum_{k=1}^3 k \right)$$

$$= 4 \left(\frac{(26)(27)}{2} - \frac{(3)(4)}{2} \right)$$

$$= 4 (351 - 6)$$

$$= \boxed{1380}$$

Ex: Evaluate $\underbrace{-3-2-1}_{-k} + 0 + \underbrace{1+2+3+\dots+25}_k$

$$= \sum_{k=1}^3 (-k) + 0 + \sum_{k=1}^{25} k$$

$$= -1 \sum_{k=1}^3 k + \sum_{k=1}^{25} k$$

$$= -1 \left(\frac{3(4)}{2} \right) + \left(\frac{25(26)}{2} \right)$$

$$= -6 + 325 = \boxed{319}$$